
Size and position invariance in the visual system

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Abstract. The encoding of simple features such as lines and angles leads naturally to size and position invariance when these features are analyzed symbolically in terms of structure and connectedness. No such analysis is possible for a spatial frequency encoding, however, as all a priori indications of pattern unity are lost. An additional transformation of spatial frequency information is therefore demonstrated that automatically derives a size invariant representation of form. The feasibility of the transform scheme in the visual system is outlined with a hypothetical processing sequence. It is found that the organization of cells in the striate cortex shows a marked similarity to an intermediate stage of the proposed sequence.

1 Introduction

The invariance in the perception of a form to changes in its position and size has always been a central question in visual research. Recent advances in computer vision (Winston 1975; Fu 1974; Duda and Hart 1973; Morofsky and Wong 1973) have demonstrated that an active structural analysis of the visual scene starting at the level of simple features—lines and angles—can arrive at such invariances. Although there are many questions as to how such an analysis could be implemented in the brain, there is no question that such demonstrations furnish a proof of existence of size and position invariance when pattern features are the point of departure. These features are, in fact, often mentioned as the initial encoding in the visual cortex (Marr 1976; Barlow et al 1972; Hubel and Wiesel 1962).

On the other hand, many authors have proposed that the striate cortex is not performing a simple feature analysis, but rather a piecewise, i.e. region by region, Fourier transform. Theoretically, the spatial variations of intensity in the input pattern are broken down into frequency components, each being characterized by an amplitude and a phase. The amplitude of a frequency component indicates the degree of presence of a sinusoidal spatial variation of luminance of a given periodicity and orientation over the entire input region; the phase of the component indicates its position in the region. The original input pattern can be reconstructed by forming the sum of all of its frequency components with the appropriate amplitudes and positions. The Fourier transform is of particular interest because of its demonstrated efficiency in pattern recognition (Andrews 1971) and this efficiency is based primarily on the position independence of the amplitude portion of the transform.

Much physiological evidence has been marshalled in support of the hypothesis of a Fourier analysis in the visual cortex (Blakemore and Campbell 1969; Maffei and Fiorentini 1973; Pollen and Ronner 1975; Maffei and Fiorentini 1977); at the same time a considerable body of research has pointed to several flaws in the concept (Burton 1976; Stromeyer et al 1973; Nachmias et al 1973; Tolhurst 1972). While the final status of the spatial frequency hypothesis awaits further evidence, it must be noted that a Fourier transform is not, by itself, invariant to size. Although the amplitude portion of the transform is independent of position, size changes of the input pattern result in equivalent, but opposite, changes in the size of the transform pattern. That is, magnification of the input pattern by a factor a produces a scale

change of all of its frequency components by a factor $1/a$; doubling the size of a square, for example, will halve all of its component frequencies.

Blakemore and Campbell (1969) have proposed that a size invariant encoding of a pattern might be generated through the analysis of the ratios of its frequency components. The ratio operation eliminates the scale factor which is common to all components. The ratios therefore remain the same as size changes. However, Blakemore and Campbell do not suggest how to determine the ratios nor how to select the components for which ratios are to be computed. When more than one pattern is present in the input, it cannot be known in advance which components belong to which pattern. The frequency components of a single pattern are, in fact, quite separate and give no indication of their actual unity in the input. (Harmonics might be considered indicators of unity on the level of pattern elements—e.g. spacing between two parallel lines, width and shape of a contour—but would be of no significance for pattern elements not sharing similar orientations.) The problem of size invariance is thus additionally complicated compared to a feature detection approach, as the type of analysis based on the connectedness of the elements in a pattern no longer applies.

Since there is no a priori pattern unity in the Fourier domain, recognition, even without size invariance, requires comparison against a known standard. The transform of the pattern being searched for is matched against the transform of the input. This template approach, often called matched filtering (Vander Lugt 1964; cf Goodman 1968), can detect the presence of a pattern of a given size at any location. It could be proposed that a template is available for every possible size of a pattern or that rapid scaling of the transform could scan across a relevant range of sizes; however, these solutions would require large amounts of memory storage and/or processing time.

Is it possible then that some other transform subsequent to or combined with the Fourier analysis could automatically check all possible transform sizes just as the Fourier itself allows a simultaneous match against all input locations? The purpose of this note is to show that such a process does exist (Cavanagh 1974; Casasent and Psaltis 1977) and that automatic size and position invariant encoding of forms is possible. While some effort will be made to outline the evidence required before a spatial-frequency-based hypothesis can be accepted, the principles of the size invariant encoding proposed here are not limited to the Fourier transform and should not be confused with it.

2 Position invariance

Consider first the spatial frequency properties of the cells in the striate cortex. Much work has shown that there are several deviations between what a Fourier transform should be and what actually appears to be happening. For example: (i) the frequency selectivity of striate cells is quite broad, up to ± 1 octave at half amplitude by some estimates (Maffei and Fiorentini 1973); (ii) the spatial analysis is only computed for small local areas, each cell representing its particular receptive field of from about $0.1 \text{ deg} \times 0.1 \text{ deg}$ visual angle to $5 \text{ deg} \times 5 \text{ deg}$ visual angle; (iii) the frequency-specific changes in cell firing rates appear to depend on the position of the input stimulus within the receptive field (Maffei and Fiorentini 1973; Pollen and Ronner 1975).

The first of these three points is not a significant problem—theoretically, a fuzzy Fourier transform contains as much information as a sharp one (see Goodman 1968, for an example of how an image may be deblurred); nevertheless, noise in the system will degrade the possible resolution of frequency components to some extent.

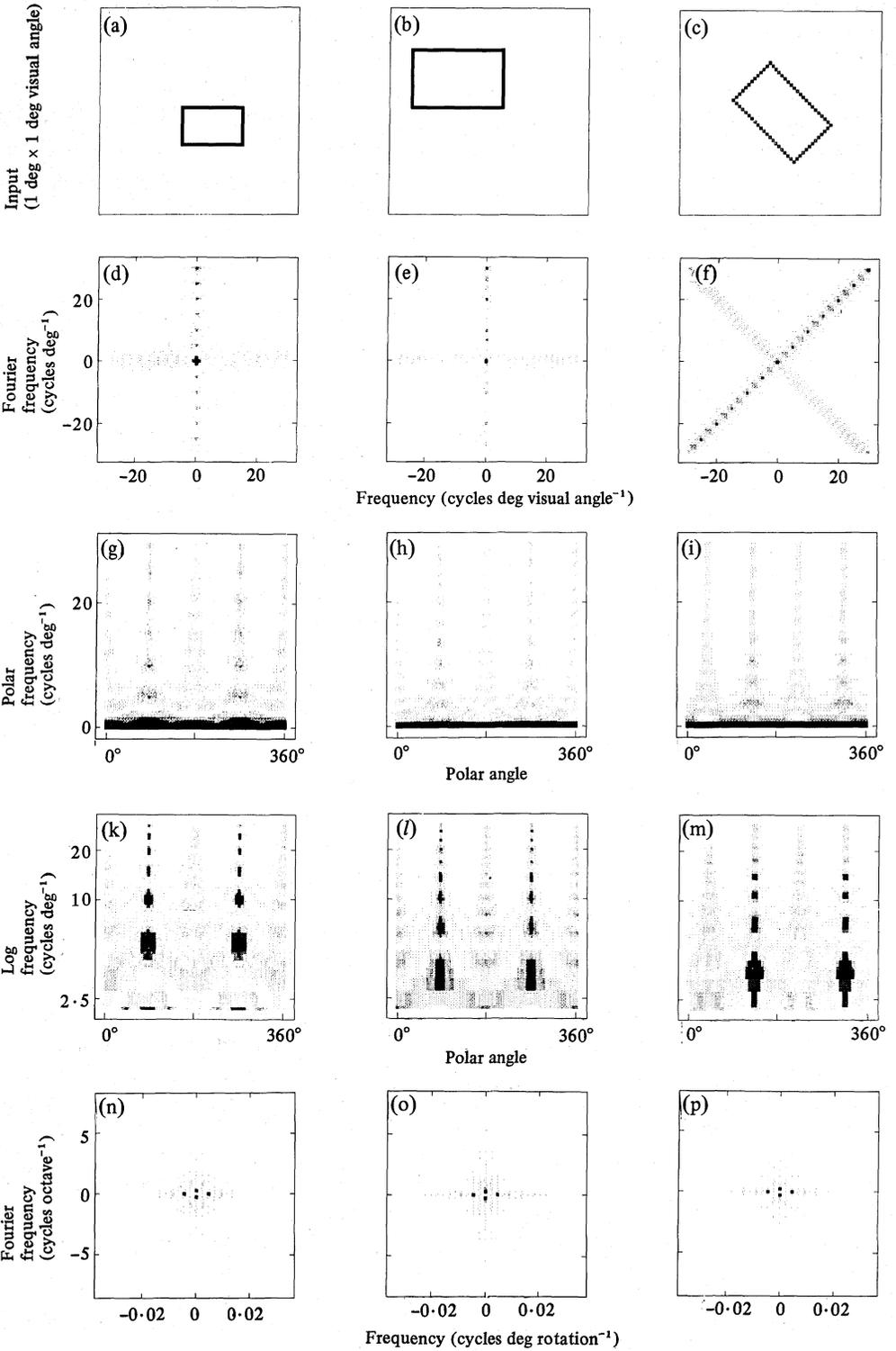
On the other hand, the second and third points are quite significant: the essential property of the Fourier transform is its position independence; it appears, however, that the analysis performed by the striate cortex is directly a function of position. For the hypothesis of Fourier analysis to have any utility, the local frequency domains must be integrated at some higher level, and an amplitude value, independent of position, must be derivable for each frequency component.

Speculation concerning the site of this additional processing must certainly include the inferotemporal cortex where experiments indicate substantial integration (Gross et al 1972). The receptive fields of cells in this area invariably include the fovea and typically extend over 40 to 60% of the entire visual field. Responses seem to be independent of the position of the stimulus within the receptive field but also seem much less dependent on spatial frequency (in fact, bar width was the parameter varied) and orientation. Although the difficulty of determining the optimum releasing stimuli for inferotemporal cells complicates the interpretation of their characteristics, this evidence nevertheless suggests that integration and position independence might be achieved here, and that, if so, additional processing has brought the encoding beyond the level of spatial frequencies. Thus, although other sites could certainly be suggested, it appears that an encoding of the input pattern could be developed in the inferotemporal region that is invariant not only to position but also to size (see Humphrey and Weiskrantz 1969; Willager and Klee 1966, for the effects of inferotemporal lesion on the perception of size). This is not to say that this would be the final representation of the visual scene to be passed on to other areas in the brain. It is more pragmatic to assume that all levels in the visual system can be accessed by higher centers to obtain different types of information—position, edges, movement, color, etc. However, in parallel with these descriptions, the inferotemporal cortex may be the locus of a form-specific encoding which has been developed from the spatial frequency information of the striate cortex.

3 Size invariance

How could a size-invariant encoding in the inferotemporal cortex be derived from the local spatial frequency analysis in the striate? The essential step is to convert the effect that a size change of the input shape has on the transform pattern to a simple position shift of an invariant transform pattern. The Fourier components of a rectangle, for example, contract or expand about the origin with changes in the size of the rectangle (figures 1a, 1b, 1d, and 1e). If the spatial frequency information can be rearranged along new dimensions so that contraction and expansion become rigid displacement of an unchanging pattern, the problem of automatic size invariance will be effectively solved. That is, size changes will have been converted to position changes at an intermediate level and a final transformation which needs only to be position independent (e.g. Fourier, Hadamard, or other; cf Kabrisky et al 1971) will complete the process.

Consider first that the axes of the spatial frequency domain are changed from the cartesian x and y to the polar representation of simply frequency and orientation (figures 1d and 1e vs figures 1g and 1h). Frequency along one axis varies from zero to plus infinity while orientation varies, orthogonally to frequency, from 0° to 360° . Now as the size of the input changes, this polar Fourier transform expands away from or contracts towards the orientation axis but always remains the same width. That is, orientation information is invariant to size changes and that invariance is now physically explicit. If the frequency axis is then changed to log frequency, the transform pattern no longer contracts or expands but remains invariant, shifting only in position in response to size changes (figures 1k and 1l). In fact, the log frequency axis automatically displays the ratios of frequency components originally mentioned



by Blakemore and Campbell (1969). [It is interesting to note that the auditory system also uses a log frequency representation (Tunturi 1952; Rose et al 1959) perhaps for the same reason.]

A final position-independent transform of this log polar frequency representation will then derive the size-invariant encoding that has been supposed to occur at the level of the inferotemporal cortex. For demonstration purposes, a Fourier transform has been used (figures 1n and 1o) although any position-independent transform would be adequate. Any arbitrary form will now have a specific transform at this level, that does not vary with size or position and can be detected with a simple template or neural filter system (Cavanagh 1975; Kohonen 1974; Anderson 1973).

While the final transform is required only to be position independent, the preceding intermediate representation must have axes of orientation and log frequency (or perhaps log width if the elementary features are assumed to be something other than sinusoidal grids) to permit size invariance. It is important to consider the possibility that the striate cortex provides not a Fourier analysis but a direct log polar frequency representation. That is, the first three steps of the transform sequence demonstrated here may perhaps be collapsed into a single step in the visual system: a log polar frequency representation directly computed by the striate cortex.

Since the log polar characteristic is only a rearrangement of the spatial frequency information, it is not necessary to look for a new set of cells in the striate cortex having different properties. The properties would again be simply frequency-specific and orientation-specific tuning. The following interpretation of data currently available is offered to show that there is merit to the suggestion of a log polar frequency arrangement of cells within local regions of the striate cortex.

4 Spatial organization of the frequency and orientation domains in the striate cortex

In the orientation domain, Hubel and Wiesel (1962) have shown that all cells along a single penetration perpendicular to the surface of the striate cortex of the cat respond maximally to approximately the same orientation. More recently, micro-electrode penetrations made tangential to the surface of the striate cortex (Albus 1975) have demonstrated that the preferred orientation of each cell changes in an orderly, continuous manner within local areas. Albus (1975) reported evidence for what he termed a bandlike structure. That is, cells that responded to the same orientation were organized in long, narrow bands on the surface of the cortex. Over short distances, these bands were often fairly straight and parallel to each other.

Figure 1. The four stages of the transform sequence are shown for a rectangle of constant proportions (3:5) at different input positions, sizes [a 50% increase from (a) to (b)], and orientations [45° rotation from (b) to (c)]. The size of the input panels (a), (b), and (c) is taken as 1 deg × 1 deg of visual angle. The images are presented on matrices of 60 by 60 with amplitude being displayed in ten equal intervals between the maximum and minimum of each image by the density of dots in each cell. The correlations (the DC terms at the origins of these transforms have been suppressed to avoid spurious correlations) between the final transforms are 0.93, (n) vs (o), 0.92, (n) vs (p). The lack of absolute size invariance results from the finite extent (4.1 octaves) of the log frequency dimension represented in the input to the final Fourier transform. Only amplitude components of the transforms have been represented. Position information is carried by the phase component of the first Fourier transform, while size and rotation information are encoded in the phase of the second Fourier. Position, size and rotation information are therefore assumed either to be analyzed through channels parallel to the form encoding sequence, or, if a physiological carrier of phase information can be demonstrated, to be derived through matched filtering processes (Vander Lugt 1964).

Preferred orientation changed in a continuous manner from one band to the next so that all orientations would be represented when traversing a cortical distance of about $900 \mu\text{m}$, perpendicular to the bands.

Experiments by Maffei and Fiorentini (1977) have replicated the findings of Hubel and Wiesel (1962) and Albus (1975). Furthermore, they extended the understanding of the spatial organization of the striate cortex to include both orientation and frequency factors. They found in penetrations parallel to the surface of the cortex that the preferred orientation of cells again changed in a continuous manner, but the preferred spatial frequency remained relatively constant. On the other hand, in penetrations perpendicular to the surface, cells maintained the same preferred orientation but changed in preferred spatial frequency. Glezer et al (1973) had also noted this latter finding but Maffei and Fiorentini collected a sufficient number of cells to indicate the possibility of continuity in the representation of spatial frequency. As the depth of penetration increased, the preferred spatial frequency first increased to a maximum and then gradually decreased again, reaching a minimum value at the deepest penetrations.

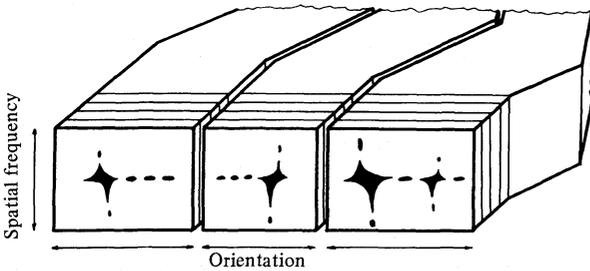


Figure 2. View of a slab of striate cortex showing a cross-section perpendicular both to Albus' (1975) isorientation bands and to the cortical surface (the top plane). Within areas where the isorientation bands are straight and parallel the local transforms will be arrayed like file cards in parallel rows across the surface of the cortex. The retinal area encoded by each local transform changes with cortical position in a retinotopic fashion. Albus' bands are not shown but would run along the cortical surface parallel to the exaggerated separations shown between the rows of local transforms. The arrangement of the local transforms, whether or not as highly ordered as that depicted here, should not influence the properties of the individual transforms to any great extent.

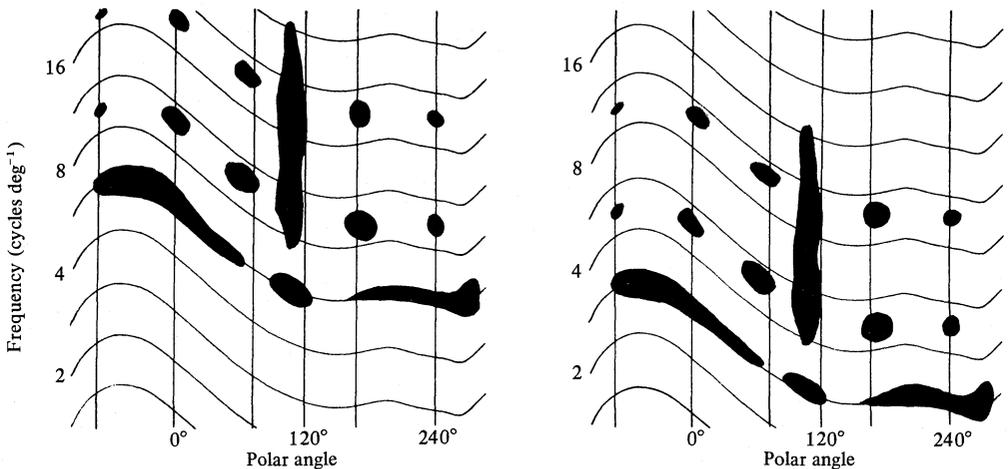


Figure 3. A log polar frequency plot that demonstrates the minimum conditions necessary for a size-invariant transform pattern: straight, parallel isorientation contours and uniform logarithmic spacing of isofrequency contours. The input pattern for the right-hand transform is the same as that for the left-hand transform, only twice as large.

The overall organization is thus pictured in figure 2. Cross-sections of the striate cortex perpendicular to Albus' (1975) isoorientation bands show orthogonal axes of spatial frequency and orientation. The cross-sections can be broken down into *local transforms*, each sufficiently wide to cover the full range of orientations [a distance of approximately 1 mm (Albus 1975)] and sufficiently deep to include a single, ordered range of spatial frequencies. Each local transform represents the spatial-frequency-by-orientation analysis for the particular region of the visual field that is covered by the receptive fields of the constituent cells.

What are the minimum requirements that would permit the conclusion that these local transforms are, in fact, log polar frequency representations having the property of size invariance? Figure 3 shows that the first of these requirements is that the isoorientation contours (lines joining cells that have identical preferred orientations) are both straight and parallel. Since cells recorded in vertical penetrations of the striate cortex (Hubel and Wiesel 1962; Maffei and Fiorentini 1977) invariably show only slight variation in preferred orientation, the isoorientation contours are all straight and perpendicular to the cortical surface. They are therefore also parallel, at least over the 1 mm width of each local transform. Notice that the isoorientation contours do not have to be linearly spaced to achieve size invariance. The data of Albus (1975) and Maffei and Fiorentini (1977) nevertheless suggest that a linear rate of change of orientation may be the case.

The remaining requirement is that the spatial frequency dimension be logarithmic and that the rate of change of log frequency along the frequency axis be the same everywhere within any given local transform. Isofrequency contours (lines joining cells of identical preferred spatial frequency) should therefore be logarithmically spaced and the spacing should not vary within a local transform. The contours, however, need be neither straight nor orthogonal to the isoorientation contours.

Maffei and Fiorentini's (1977) data indicate that the representation of spatial frequency as function of depth, although continuous, is not monotonic. That is, there are two representations of the frequency axis, first increasing and then decreasing⁽¹⁾. If the proposal of a log polar frequency encoding is correct, either one or both of these frequency representations must be logarithmic. Maffei and Fiorentini did not collect enough cells to permit this possibility to be either accepted or rejected. Their data did indicate, however, that the isofrequency contours may be both fairly straight and orthogonal to the isoorientation contours.

There is indirect evidence that the spatial frequency tuned cells are distributed logarithmically. Campbell et al (1970) showed that the just noticeable change in the spatial frequency of a grating is a constant percentage of its frequency. This constant percentage j.n.d. is classical evidence of an underlying logarithmic dimension (Fechner 1860).

(1) The first of these possible axes covers the upper layers of the striate cortex, II and III. Although it is difficult to judge from Maffei and Fiorentini's (1977) limited data sample, preferred frequency appears to increase with depth as these layers are traversed. These upper layers project together to areas 18 and 19 (Martinez-Millan and Holländer 1975; Spatz et al 1970) and from there to the inferotemporal cortex (Gross et al 1972). The layers making up the second possible axis, that is IV, V and VI, across which preferred frequency progressively decreases (Maffei and Fiorentini 1977), all send their projections to separate subcortical areas (Lund et al 1975). The transform sequence proposed here requires that a local transform that is arrayed in depth across these layers be treated as a single image in subsequent processing. Although the projections eventually return through various routes to areas 18, 19, and the inferotemporal cortex, the separate routes taken by information from the different layers makes it difficult, as one reviewer has pointed out, to suppose that these lower layers could act functionally as a single unit.

In addition, it has been suggested that the range of frequencies for which a given cell receives excitatory input is quite broad but that the tuning of the cell's output is sharpened considerably by lateral inhibition from adjacent cells tuned to slightly different values (Tolhurst 1972). The shape of a cell's tuning curve should therefore give some indication of the range of frequencies covered within the lateral inhibition radius (assumed to be constant at all positions) at the particular orientation and frequency to which the cell is tuned.

Studies in the visual cortex of the cat (Maffei and Fiorentini 1973, 1977; Glezer et al 1976) show that the cells appear to have constant ratio bandwidths. In other words, the bandwidth of the cell's tuning curve can be described by the number of octaves separating the upper and lower spatial frequencies at which the cell's response is one-half its maximum level. This bandwidth maintains a fairly steady value—1 to 2 octaves—independently of the cell's preferred frequency. Furthermore, Maffei and Fiorentini (1973, 1977) and Glezer et al (1976) did not report any variation of the bandwidth as a function of the cell's preferred orientation. These results imply that cells within the lateral inhibition radius of the cell being tested must cover a fixed number of octaves, i.e. *log units*, of spatial frequency independently of the frequency and orientation to which the cell is tuned.

If lateral inhibition and bandwidth are related as suggested, then isofrequency contours must be uniformly, logarithmically spaced at all frequencies and orientations. This property, in combination with the straight parallel isorientation contours, is sufficient to provide a local size-invariant encoding which shifts in position but not form when the size of an input pattern is changed.

The work of Albus (1975) and Maffei and Fiorentini (1977) has demonstrated the presence of local transforms with axes of orientation and frequency—findings that provide direct support for the hypothesis of a log polar frequency representation. On the other hand, the evidence concerning logarithmic nature of the frequency dimension, although favorable, is of a strictly indirect nature. The arguments given here can therefore be considered only hypothetical. It is nevertheless interesting to note that the organization of the frequency and orientation dimensions seen experimentally (Albus 1975; Maffei and Fiorentini 1977) would not support a local Fourier transform hypothesis.

5 Rotational properties

On the basis of the arguments presented in the preceding section, it is not unreasonable to propose a direct local log polar frequency analysis in the striate cortex that might lead, after integration and a further position independent transform, to a size-invariant encoding in the inferotemporal cortex. The spatial frequency hypothesis is thus at no disadvantage in terms of its final utility when compared to a feature detection scheme. In fact, it would appear to have some advantages. First, the mechanisms of the size-invariant encoding are physiologically explicit and thus eventually testable. Second, active context-sensitive processing, which is still required to interpret such factors as depth, ambiguous figures, and partially hidden objects to name only a few, would now have access to information on the form level which is unavailable to a simple feature-based encoding.

As an independent test of the processing sequence proposed here, it is informative to consider what effects are produced by the rotation of the input pattern. It will be assumed that the isofrequency contours of the log polar frequency plot form straight parallel lines and that the isorientation contours are linearly spaced. Any departures from these two conditions will lead to degradation of the rotation characteristics described here. These two conditions are seen in figures 1k, 1l, and 1m, where rotation of the input produces a linear shift of the transform pattern. Note, however,

that the orientation axis contains a break in its representation of orientations. Such breaks are seen experimentally (Albus 1975; Maffei and Fiorentini 1977) as abrupt reversals in the direction of the representation and can be assumed to be the borders of the local transforms. While the position of the discontinuity given here (0°) is arbitrary, the requirement of parallel isorientation contours makes the discontinuity a necessary adjunct to spatial-frequency-based size-invariant encoding. Unlike the shift produced by size, then, a shift along the orientation axis pushes part of the pattern off the edge. The displaced portion reappears at the opposite edge; at 180° rotation, half of the pattern has been lost on one side and replaced on the other. It is apparent that the encoding of the rotated pattern will resemble the original less and less up to 180° , when 50% of the pattern remains continuously represented. Fidelity would return to 100% at 360° rotation. This implies that recognition should be very broadly tuned and that, while being most impaired at 180° , recognition would certainly still be possible at that orientation.

This very broadly tuned sensitivity is seen experimentally. In reaction-time experiments, for example, a mirror image discrimination takes 400 ms longer for an upside-down test letter than for an upright letter (Cooper and Shepard 1973). Naming an upside-down letter takes only 50 ms longer than naming an upright letter (Corballis and Zbrodoff 1975). In both of these cases, errors are quite low. When errors are measured for recognition of complex stimuli at various orientations, performance again deteriorates as test-figure rotation approaches 180° but is always above chance (Arnoult 1954; Rock 1974). The observed orientation tuning of form recognition therefore appears to be much greater than the $\pm 20^\circ$ that is reported for orientation-specific cells in the striate cortex (Campbell et al 1968). A simple Fourier encoding is necessarily limited to the orientation selectivity of frequency- and orientation-specific cells; the orientation tuning of the transform sequence demonstrated here is not limited by the selectivity of striate cells and appears to resemble the broad tuning characteristic of human vision.

As a final aside, note that if the last transform of the sequence proposed here were actually Fourier, or Fourier-like, one of its axes would code simply for angles, independently of orientation. That is, straight lines at the input also produce straight lines of frequency components which cross, at the origin, at the same angle. Figure 1 demonstrates this for lines at a 90° angle. The polar transform turns these into vertical lines whose spacing varies directly with the input angle. This spacing is encoded as a specific horizontal spatial frequency (with appropriate harmonics) by the second Fourier transform. Figures 1n, 1o, and 1p thus show strong peaks on their horizontal axes, which represent the 90° angles of the input rectangles, independently of orientation. Smaller angles would produce peaks farther out on this axis.

The possible existence of an angle axis allows interesting conjectures concerning lateral inhibition between cells along this dimension. Such a mechanism might contribute to tilt illusions and, in particular, to so-called indirect effects in these illusions (O'Toole and Wenderoth 1977).

6 Summary

In conclusion, the purpose of this note has been to demonstrate the possibility that a spatial frequency analysis can lead to a size-invariant encoding of visual patterns. The validity of the transform sequence presented here cannot be tested with the data currently available, however. Before the proposed mechanisms can be seriously considered it must be shown both that size-invariant encoding does occur either in the inferotemporal cortex or elsewhere, and that this encoding depends on the output of the frequency- and orientation-specific cells of the striate cortex.

It is assumed that such an encoding would proceed in parallel with other, perhaps feature-based, analyses of the visual input.

If the visual system does use a transform approach to achieve size invariance, it may or may not be similar to the sequence presented here. Nevertheless, it is difficult to ignore the similarity between the log polar frequency representation required for the proposed transform and the actual organization of frequency and orientation axes in the striate cortex. Unless some other purpose can be demonstrated for this specific organization, it is not unreasonable to assume that it exists in order to process shape information in some manner similar to that outlined here.

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